B.A./B.Sc. 2nd Semester (General) Examination, 2022 (CBCS)

Subject: Mathematics

Course: BMG2CC1B & Math-GE2

(Differential equations)

Time: 3 Hours | Full Marks: 60

The figures in the margin indicate full marks.

Candidates are required to write their answers in their own words as far as practicable.

[Notation and Symbols have their usual meaning]

1. Answer any two questions 10×2 = 20

(a) Find the family of curves orthogonal to $y^2 - x^2 = a^2$, where a is a constant. [2]

(b) Show that the Wronskian of the functions x^2 and $x^2 log x$ is non-zero. Can these functions be independent solutions of an ordinary differential equation? [2]

(c) Find the integrating factor of the differential equation $\frac{dy}{dx} + \frac{1}{1+x^2} y = \frac{e^{tan^{-1} x}}{1+x^2}$. [2]

(d) Solve: $\frac{2xdx}{y^3} + \frac{y^2-3x^2}{y^4} dy = 0$. [2]

(e) Show that the equation $(2x + y^2 + 2xz)dx + 2xydy + x^2 dz = 0$ is integrable. [2]

(f) Solve:$p^2 - p(e^x + e^{-x}) + 1 = 0, where\ p = \frac{dy}{dx}$. [2]

(g) Determine the type of the equation $2\frac{\partial^2 z}{\partial x^2} - 3\frac{\partial^2 z}{\partial x \partial y} + 5\frac{\partial^2 z}{\partial y^2} = 0$. [2]

(h) Form the partial differential equation by eliminating the arbitrary functions h and k from the equation $y = h(x - at) + k(x + at)$. [2]

(i) Find integrating factor (I.F) of $(x^2 y - 2xy^2)dx + (3x^2 y - x^3)dy = 0$. [2]

(j) Evaluate :$\frac{1}{(D-2)^2} x^3 e^{2x}$, where $D \equiv \frac{d}{dx}$. [2]

(k) If the roots of the auxiliary equation of an ODE are 0,1,1,2$\pm$3i find its complementary function. [2]

(l) Evaluate: $\frac{1}{D^2-9} e^{e^x}$,where$D \equiv \frac{d}{dx}$. [2]

(m) Solve: $(D - 1)^2(D^2 + 2)^2$y=0,where $D \equiv \frac{d}{dx}$. [2]

(n) Define the principle of superposition. [2]

(o) Verify with reason that $\sin x$ and $\cos x$ are two linearly independent solution of $\frac{d^2y}{dx^2} + y = 0$. [2]

2. Answer any four questions $4\times5 = 20$

(a) Obtain the general and singular solution of $y = px + \frac{a}{p}$, where $p = \frac{dy}{dx}$ and a is a constant. [5]

(b) Solve: $(x^2D^2 + 3xD + 1)y = x\,logx$, $D \equiv \frac{d}{dx}$. [5]

(c) Solve: $\frac{dx}{dt} + 4x + 3y = t,\ \frac{dy}{dt} + 2x + 5y = e^t$. [5]

(d) Using Charpit's method find complete integral of $xp + 3yq = 2(z - x^2q^2)$, $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$. [5]

(e) Solve the initial value problem [5]
$(2x\cos y + 3x^2y)\,dx + (x^3 - x^2\sin y - y)dy = 0$ with $y(0) = 2$.

(f) Solve the simultaneous equations: $\frac{dx}{dt} + y = \sin t, \frac{dy}{dt} + x = \cos t$. Given that $x = 2$ and $y = 0$ when $t = 0$. [5]

3. Answer any two questions $2\times10 = 20$

(a) (i) Solve by the method of variation of parameters: $\frac{d^2y}{dx^2} + a^2y = \tan ax$. [5]

(ii) Solve: $y - x\frac{dy}{dx} = 2(1 + x^2\frac{dy}{dx})$; given $y(1) = 1$. [5]

(b) (i) Solve: $x(y - z)p + y(z - x)q = z(x - y), p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$. [4]

(ii) Eliminate the arbitrary function f from $f(x - y, y/x^2) = 0$. [3]

(iii) Find the partial differential equation of the set of all right circular cones whose axes coincide with z-axis. [3]

(c) (i) Find the integral surface of $x^2p + y^2q + z^2 = 0, p = \frac{\partial z}{\partial x},\ q = \frac{\partial z}{\partial y}$ which passes through the hyperbola $xy = x + y, z = 1$ [5]

(ii) Solve $\frac{1}{D^2+a^2}\mathrm{x}^2\cos ax$, where $D \equiv \frac{d}{dx}$. [5]

(d) (i) Solve : [5]

$$\frac{d^2x}{dt^2} - 5\frac{dx}{dt} + 6x + 2e^{2t} + 3e^t = 0.$$

(ii) Solve :$(y - px)(p - 1) = p,\ where\ \ p = \frac{dy}{dx}$. [5]